

機電所博士生資格考試題 2008

工程數學(Advanced Engineering Mathematics)

每題二十分，總分一百分。

1. Diagonalize the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Find the matrix $\mathbf{P}$ that diagonalizes $\mathbf{A}$ and the diagonal matrix $\mathbf{D}$ such that $\mathbf{D} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$.

2. L^{-1} is the inverse Laplace transform, evaluate

$$(a) \quad L^{-1}\left\{\frac{s-1}{s^2+4}\right\}, \quad (b) \quad L^{-1}\left\{\frac{e^{-s}}{(s+1)^2}\right\}.$$

3. Solve the Bernoulli's equation

$$x^2 \frac{dy}{dx} + y^2 = xy.$$

4. Solve the Laplace's equation: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $0 < x < a$, $0 < y < b$, subject to

$$u(x,0)=u(x,b)=0, \quad 0 < x < a; \quad u(0,y)=0, \quad u(a,y)=T \text{ (a constant)}, \quad 0 < y < b.$$

5. Expand $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 2, & 0 \leq x < \pi \end{cases}$ in a Fourier series.

國立虎尾科技大學 機電所 博士班資格考

- 1 Determine the eigenvalue and eigenvector of the problem

$Ax = \lambda Bx$, where

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

And verify that the eigenvectors are orthogonal relative to both A and B.

- 2 Solve for $x^3 y''' + x^2 y'' - 2xy' + 2y = x^3 \ln x$

- 3 $\vec{F} = \left(\frac{-y}{x^2 + y^2} \right) \vec{i} + \left(\frac{x}{x^2 + y^2} \right) \vec{j}$, find

a) $\nabla \times \vec{F}$

b) $\oint_C \vec{F} \cdot d\vec{r}$

where C is any simple closed curve, and $\vec{r}$ is the position vector.

- 4 Please solve the following boundary value problem

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad (0 < x < L, t > 0)$$

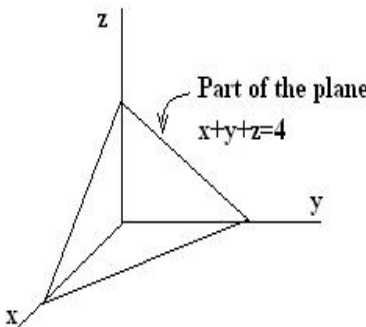
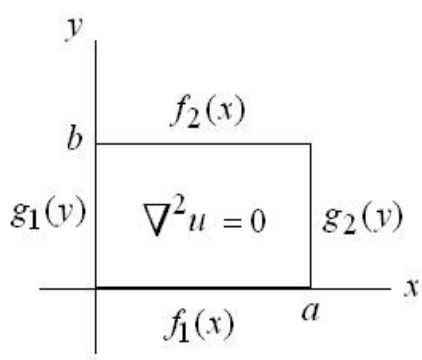
with

$$\frac{\partial T}{\partial x}(0, t) = 1$$

$$\frac{\partial T}{\partial x}(L, t) = 0$$

and

$$T(x, 0) = 0$$

1. (15%)	A force $\vec{F}(x, y, z) = x^2 \vec{i} - zy \vec{j} + x \cos(z) \vec{k}$ move a particle along the path C given by $x = t^2$, $y = t$, $z = \pi t$ for $0 \leq t \leq 3$. The initial point is P: (0,0,0) and the terminal point of Q is Q: (9,3,3π). Suppose we want the work done in moving the particle along this path from Q to P.	
2. (15%)	Evaluate $\iint_{\Sigma} z d\sigma$ if Σ is the part of the plane $x+y+z=4$ lying above the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 1$. 	
3. (25%)	Solve the system of linear differential equations $\underline{X}' = \underline{A}\underline{X}$, where $\underline{A} = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \quad \underline{X} = [x_1, x_2, x_3, x_4], \text{ and } \underline{X}' = \frac{d\underline{X}}{dt}.$	
4. (20%)	Solve the system of linear differential equations and initial conditions for the functions x and y: $x'' - 2x' + 3y' + 2y = 4 \quad \text{and} \quad x'(0) = x(0) = y(0) = 0$ $2y' - x' + 3y = 0$	
5. (25%)	Find the solution of partial differential problem as shown in the following Figure. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0; \quad \begin{matrix} 0 \leq x \leq a \\ 0 \leq y \leq b \end{matrix}$ $u(x,0) = f_1(x), \quad u(0,y) = g_1(y),$ $u(x,b) = f_2(x), \quad u(a,b) = g_2(y),$	

學號：_____ 姓名：_____

1. 解： $2xy' + 3x^2y^2 + y = 0$

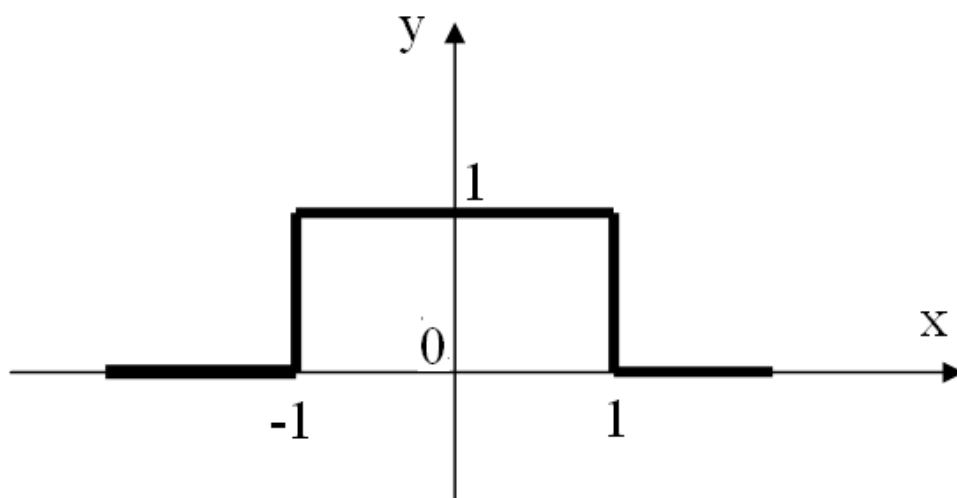
2. 解： $y'(t) = \int_0^t y(x) \cos(t-x) dx, y(0) = 1$

3. 已知 $A = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$, 求 e^{At}

4. 計算： $\oint_C (3y - e^{\sin x}) dx + (7x + \sqrt{y^4 + 1}) dy$,

其中 C 是圓 $x^2 + y^2 = 9$.

5. (1) 計算下圖的傅立葉積分式：



(2) 求 $\int_0^{\infty} \frac{\sin t}{t} dt$

1. 解下列微分方程式：(20%)

$$y'' - 3y' + 2y = e^{3x}$$

2. 試求下列函數之拉普拉斯轉換(Laplace Transform) (20%)

(a) $f(t) = e^{3t}t + e^{-3t}t^2$

(b) $f(t) = e^{3t}(\cos 2t + \sin 2t)$

3. 請以高斯消去法(Gauss Elimination)解下列聯立方程式(20%)

(a)
$$\begin{cases} x_1 + x_2 = 3 \\ 2x_1 - x_2 = 0 \end{cases}$$

(b)
$$\begin{cases} 2x_1 + x_2 - x_3 = 5 \\ x_1 - 3x_2 + x_3 = 2 \\ x_1 + 3x_2 - 3x_3 = 0 \end{cases}$$

4. 解下列微分方程式：(20%)

$$y' - \frac{2}{x}y = x^2 e^{-x}$$

5. 試求下列函數之拉普拉斯反轉換：(20%)

(a) $F(s) = \frac{s}{s^2 + s - 6}$

(b) $F(s) = \frac{s}{(s+2)(s-3)^2}$

1. Solve I.V.P.: $y'' - 4y' + 5y = 21e^{2x}$, $y(0) = 1$, $y'(0) = 0$. (25%)
2. Solve I.V.P. using Laplace transform: $y'' + 4y = f(t)$, $y(0) = 0$, $y'(0) = -1$, where $f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$. (25%)
3. Let $F(x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$ and Σ be the piecewise smooth closed surface consisting of the surface Σ_1 of the cone $z = \sqrt{x^2 + y^2}$ for $x^2 + y^2 \leq 1$, together with of flat cap Σ_2 consisting of the disk $x^2 + y^2 \leq 1$ in the plane $z = 1$. Verify the Gauss's divergence theorem, i.e. $\iint_{\Sigma} F \cdot N d\sigma = \iiint_V \nabla \cdot F dV$. (25%)
4. Find the eigenvalues of matrix A and the corresponding eigenvectors. (25%)

$$A = \begin{bmatrix} -3 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

國立虎尾科技大學 機械與機電工程研究所

100 學年度第 2 學期 博士班資格考 【工程數學】

1. 試求出下列微分方程式之解：

10% (a) $y'' - 2y' - 3y = 2\sin^2 x$

10% (b) $y' = 2 + \left(\frac{y}{x}\right)^2$

15% (c) $y'' + 5y' + 6y = f(t); y(0) = 0, y'(0) = -4$

$$f(t) = \begin{cases} t^2, & \text{if } 0 \leq t < \pi \\ \cos t, & \text{if } t \geq \pi \end{cases}$$

2. 已知 $f(x) = \frac{1}{2}x^2$ for $-\pi \leq x \leq \pi$

14% (a) 試求出其 Fourier series

6% (b) 並以(a)之答案求出 $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2} = \frac{-1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} + \cdots = ?$

3.

5% (a) 已知 $f(x) = H(x)e^{-ax}$ ($a > 0$), $H(x)$ 為 Heaviside step function, 試求 $f(x)$ 之 Fourier transform, $F[f(x)] = ?$

10% (b) 試求 $F[e^{-a|x|}] = ?$ 即 $e^{-a|x|}$ 之 Fourier transform。

10% (c) 試求 $F^{-1}[e^{-|w|} \cos w] = ?$ 即 $e^{-|w|} \cos w$ 之 Fourier inverse transform。

4. 已知 $A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix}$

5% (a) 試求 A^{-1} (A 之反矩陣)。

5% (b) 試求 A 之 eigenvalue 及 eigenvector。

10% (c) 試求 $A^{30} = ?$

國立虎尾科技大學 102 學年度第一學期博士班資格考試題

所別：動力機械工程系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 題，任選 題作答，每題 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止/可 使用計算機

solving the following differential equation。

$$(1) 2xyy' + (x-1)y^2 = x^2e^x$$

$$(2) x^2y'' - 2xy' + 2y = 16x^3\cos x$$

(3) using Laplace transform to solve the following differential equation。

$$y'' + 3y' + 2y = 4t^2 \text{ if } 0 < t < 1 \text{ and } 8 \text{ if } t > 1;$$

$$y(0) = 0, y'(0) = 0$$

(4) find the following function $f(x)$ of the Fourier integral。

$$f(x) = e^{-|x|} (-\infty < x < \infty)$$

(5)

(a). find the following function $f(x)$ of line integral $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ using the region R boundary

C to calculate by counter clockwise

$\mathbf{F} = [6y^2, 2x-2y^4]$, R is the square of the four points $\pm(2, 2)$ and

$\pm(2, -2)$ 。

(b). calculate the above problem by using the green threom to find integral $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ 。

國立虎尾科技大學 102 學年度第二學期博士班資格考試題

所別：動力機械系機械與機電工程博士班

第 1 頁 共 2 頁

科目：工程數學

注意事項：

- (1) 本試題共有 4 題，依題配分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機，Close Book。

25% (1) Please solve the following differential equation

11% (a) $2yy' + y^2 \sin x = \sin x$, $y(0) = \sqrt{2}$

14% (b) $x^2 y'' - 2y' + y = x \ln|x|$

$$y(1) = 1, \quad y'(1) = -4$$

15% (2) Please use laplace transform to solve the following equation

$$y'' + 9y = f(t) \quad f(t) = \begin{cases} 0 & 0 \leq t \leq \pi \\ \cos t & t \geq \pi \end{cases}$$

40% (3)

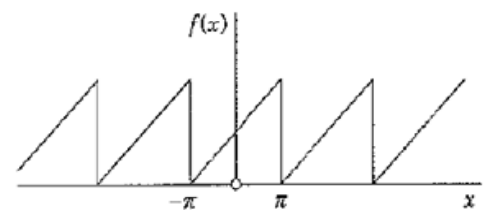
15% (a) find the Fourier series of the Function (Fig.1)

$$f(x) = f(x + 2\pi)$$

5% (b) using the above the answer, find the following series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \cdots (-1)^n \frac{1}{2n+1} \cdots = ?$$

20% (c) Solve the following differential equation



(Fig.1)

$$y'' - 4x = f(x) \quad , \quad f(x) \text{ is the function of Fig.1}$$

$$20\%(4) \quad [A] = \begin{bmatrix} -6 & 11 & 3 \\ 4 & 1 & 3 \\ -4 & 10 & 8 \end{bmatrix}$$

8%(a) find the eigenvalue and eigenvector of $[A]$, one of eigenvalue of $[A]$ is

$$\lambda_1 = 2$$

12%(b) find $[A]^{20} = ?$

國立虎尾科技大學 103 學年度第一學期博士班資格考試題

所別：動力機械系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

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- (3) 禁止使用計算機，close book

1. Solve the following problems.

a. $y'' - 6y' + 9y = 5e^{3x}$; b. $x^2 y'' - 4xy' + 6y = x^4 e^x$; $y(2) = 2, y'(2) = 7$

2. Solve the following problems by the Laplace Transform

a. $y'' + 4y = f(t); f(t) = \begin{cases} 0 & \text{for } t < 3 \\ t & \text{for } t \geq 3 \end{cases}; y(0) = y'(0) = 0;$

b. $y'' + 2y' + 2y = \delta(t - 3); y(0) = y'(0) = 0;$ where $\delta(t)$ is the Dirac Delta function.

3. Find the Fourier series of the following functions.

a. $f(x) = x$ for $-\pi \leq x \leq \pi;$

b. $f(x) = \begin{cases} 0 & \text{for } -3 \leq x \leq 0 \\ x & \text{for } 0 \leq x \leq 3 \end{cases}$

4. Solve the following Nonhomogeneous systems of linear equations

$$x_1 - x_3 + 2x_4 + x_5 + 6x_6 = -3$$

a. $x_2 + x_3 + 3x_4 + 2x_5 + 4x_6 = 1$

$$x_1 - 4x_2 + 3x_3 + x_4 + 2x_6 = 0$$

b.
$$\begin{pmatrix} 2 & 1 & -11 \\ -5 & 1 & 9 \\ 1 & 1 & 14 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -6 \\ 12 \\ -5 \end{pmatrix}$$

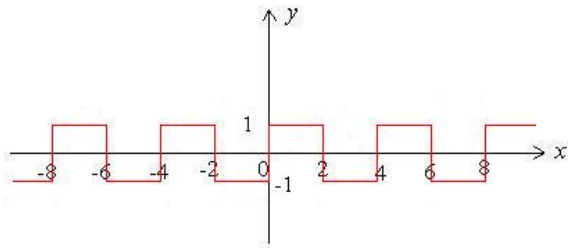
國立虎尾科技大學 103 學年度第二學期博士班資格考試題

所別：動力機械系機械與機電工程博士班本試題

科目：工程數學

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- (3) 禁止計算機，本試題為 close book 考試

1.	Solve the following Ordinary Differential equations: (a). $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$, (b) $y'' - 2y' + y = x^2 e^x$, and (c) $x^2 y'' - 5xy' + 8y = 2\ln(x) + x^3$
2	Expand $f(x) = \begin{cases} 1, & 0 \leq x \leq 2 \\ -1, & -2 \leq x \leq 0 \end{cases}$ and $f(x+4) = f(x)$ into the Fourier series. Then, find the values of the following series: (a) $\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)}$ and (b) $\sum_{m=1}^{\infty} \frac{1}{(2m-1)^2}$. 
3	Solve $y'' + y - 4 \int_0^t y(\tau) \sin(t - \tau) d\tau = e^{-2t}$, where $y(0) = 1$ and $y'(0) = 0$.
4	Solve the system of Linear differential equations $\begin{cases} x_1'' + 10x_1 - 4x_2 = 0 \\ 4x_1 + x_2'' + 4x_2 = 0 \end{cases}, \quad \text{where the initial conditions are given as } \begin{cases} x_1(0) = 0 & x_1'(0) = 1 \\ x_2(0) = 0 & x_2'(0) = -1 \end{cases}$
5	Prove the matrix $\underline{A} = \begin{bmatrix} 0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ 1 & 0 & 0 \\ 0 & \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix}$ is orthogonal.

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科目：工程數學

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- (3) 禁止使用計算機且 close book

1. Solve the initial value problem (IVP): $y'' - 3y' + 2y = x + 2$; $y(0) = 0$, $y'(0) = 0$.

2. Find the initial value problem (IVP): $x^2 y'' - 5xy' + 8y = 2\ln(x)$; $y(1) = 0$, $y'(1) = 0$.

3. Use "**Laplace transform**" to solve the initial values problem (IVP): $y'' + 4y = f(t)$; $y(0) = 0$, $y'(0) = 0$, in which $f(t) = \begin{cases} 0 & \text{for } t < 3 \\ t & \text{for } t \geq 3 \end{cases}$.

4. Find the eigenvalues and the corresponding eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}.$$

5. A function $f(x) = x^2$ is defined on $[-\pi, \pi]$. Find its Fourier series on the defined interval.

國立虎尾科技大學 104 學年度第 2 學期博士班資格考試題

系別：動力機械系機械與機電工程博士班

科目：工程數學

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- (3) 禁止使用計算機且 close book

1. Solve the initial value problem (IVP) : $y'' + 4y' - 2y = 2x^2 - 3x + 6$; $y(0) = -9$, $y'(0) = 0$.

2. Find the initial value problem (IVP): $x^2 y'' - 4xy' + 6y = \ln(x^2)$; $y(1) = \frac{5}{18}$, $y'(1) = 0$.

3. Use "Laplace transform" to solve the initial value problem (IVP): $y'' - 2y' + 5y = 1 + t$; $y(0) = 0$, $y'(0) = 4$.

4. Solve the equations of system by any matrix technique.

$$\begin{cases} 3x_1 + 2x_2 + x_3 = 7 \\ x_1 - x_2 + 3x_3 = 3 \\ 5x_1 + 4x_2 - 2x_3 = 1 \end{cases}$$

5. Expand $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \leq x \leq \pi \end{cases}$ in a Fourier series.

國立虎尾科技大學 105 學年度第二學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第一頁 共一頁

科目：工程數學

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- (3) 禁止使用計算機 且 close-book exam

1. Solve the initial value problem (IVP) : $y'' + y' - 6y = 2x$; $y(0) = 1$, $y'(0) = 0$.

2. Solve the initial value problem (IVP) : $x^2 y'' + 10xy' + 8y = x^2$; $y(1) = 0$, $y'(1) = 0$.

3. Use “**Laplace transform**” to solve the initial values problem (IVP): $y'' - 3y' + 2y = e^{-4t}$;
 $y(0) = 1$, $y'(0) = 5$.

4. Find the eigenvalues and the corresponding eigenvectors for the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$.

5. Find the half-range cosine and sine expansions of the given function $f(x) = \begin{cases} 1 & \text{for } 0 < x < 1/2 \\ 0 & \text{for } 1/2 \leq x < 1 \end{cases}$.

國立虎尾科技大學 106 學年度第一學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第一頁 共一頁

科目：工程數學

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- (3) 禁止使用計算機 且 close book

1. (a) Solve the initial value problem (IVP): $y' + \frac{1}{x}y = -\frac{2}{x}$; $y(1) = 1$.
(b) Solve the initial value problem (IVP): $4xdx + 9ydy = 0$; $y(2) = 0$.
2. Solve the initial value problem (IVP) : $y'' - 2y' + y = 2\sin(x)$; $y(0) = 0$, $y'(0) = 0$.
3. Use “**Laplace transform**” to solve the initial values problem (IVP): $y'' - 5y' + 6y = H(t-1)$; $y(0) = 0$, $y'(0) = 1$. (Note: $H(t-1)$ is a unit step function)
4. Find the rank of the given matrix $A = \begin{bmatrix} 1 & -2 & 3 & 4 \\ 1 & 4 & 6 & 8 \\ 0 & 1 & 0 & 0 \\ 2 & 5 & 6 & 8 \end{bmatrix}$.
5. Expand $f(x) = x^2$, $0 < x < L$,
 - (a) in a cosine series
 - (b) in a sine series.

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系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 8 題，每題 12.5 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 可使用計算機 close book

1. Solve(求解微分方程式)

$$y' = \frac{y(y-2)}{x(y-1)}$$

2. Solve(求解微分方程式)

$$\sin y \, dx + (x \cos y - 2y) \, dy = 0$$

3. Solve(求解微分方程式)

$$y'' + 4y' + 7y = 0$$

4. Solve(求解微分方程式)

$$x^3 y''' - 3x^2 y'' + 7xy' - 8y = 0$$

5. Known(已知)

$$L\{\sin at\} = \frac{a}{s^2 + a^2}, \quad (s > 0)$$

Evaluate(求) $L\{t \sin at\}$

6. By the elementary row operations(使用基本列運算), to evaluate the inverse

matrix of $\begin{bmatrix} 1 & 3 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (求取反矩陣)。

7. Solve the eigenvalues and eigenspaces of $\begin{bmatrix} 1 & 3 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (試求矩陣的特徵值與特徵空間)。

8. Solve(求解方程組)

$$\begin{aligned} x' &= x + 4y - 4t^2 - 3, & x(0) &= 2 \\ y' &= x + y - t^2 + 2t - 3, & y(0) &= 3 \end{aligned}$$

國立虎尾科技大學 107 學年度第一學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 可使用計算機 close book

1. (a) Solve the differential equation(求解微分方程式)

$$dx + (3x - e^{-2y})dy = 0$$

(b) Solve the differential equation(求解微分方程式)

$$y'''' - y'' = 0$$

2. To evaluate (求解) $L^{-1}\left(\ln \frac{s-a}{s-b}\right)$, where a and b are real numbers.

3. Determine all eigenvalues and eigenspaces (求矩陣的特徵值與特徵空間) of

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

4. Solve the system (求解方程組)

$$\begin{aligned} x' &= x + 4y - 4t^2 - 3, & x(0) &= 2 \\ y' &= x + y - t^2 + 2t - 3, & y(0) &= 3 \end{aligned}$$

5. Expand the square wave function defined below in Fourier series (試求下列方波函數之 Fourier 級數展開).

$$f(x) = \begin{cases} 0, & x \in (-\pi, 0) \\ 4, & x \in (0, \pi) \end{cases}$$

$$f(x + 2\pi) = f(x)$$

國立虎尾科技大學 107 學年度第二學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，任選○題作答，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) **禁止 使用計算機**

1. Given

$$y'' + 2y' + 2y = r(t) \quad \begin{cases} r(t) = 10 \sin 2t & \text{if } 0 < t < \pi \\ r(t) = 0 & \text{if } t > \pi \end{cases} \quad \begin{cases} y(0) = 1 \\ y'(0) = -5 \end{cases}$$

- (1) Solve the initial value problem above using Laplace Transfer method.
- (2) What mechanical system can apply above differential equation?

2. At point P in O_{xi} coordinate axes the stress tensor is

$$\sigma_{ij} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

Find values of principal stresses (i.e, eigen values) and the directions associated with them (i.e., principal axes)

3. (1) Please find Fourier Series of $f(t)$ below

$$f(t) = \begin{cases} -1 & \text{if } -2 < t \leq 0 \\ 1 & \text{if } 0 < t \leq 2 \end{cases} \quad T=4$$

(2) Please find Fourier Transfer of $f(x)$, "a" is any constant.

$$f(x) = e^{-a|x|}$$

4. Please solve differential equation below

(1) $(2x - 4y + 5)y' + x - 2y + 4 = 0$

(2) $xy' + 2y = 4e^{x^2}$

5. Find reverse Matrix of A (i.e., A^{-1})

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

- (1) Using Gauss-Jordan Elimination (2) using Determinants (i.e., Det A)

國立虎尾科技大學 108 學年度第一學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第一頁 共一頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 大題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機 且 close book

1. Solve I.V.P. $y'' + 7y' + 12y = 12x + 19; y(0) = 1, y'(0) = 0$.
2. Use Laplace transform to solve I.V.P. $y'' - 3y' - 4y = H(t - 1); y(0) = 2, y'(0) = 4$. (Note: $H(t-1)$ is a unit step unction)
3. Diagonalize the matrix $A = \begin{bmatrix} -2 & 7 & 3 \\ 6 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$.
4. Let $f(x) = \begin{cases} 0, & -\pi < x \leq 0 \\ \pi - x, & 0 < x \leq \pi \end{cases}$ and $f(x) = f(x + 2\pi)$. Expand $f(x)$ in Fourier series.
5. Let a vector field $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$.
 - (a) Show $\vec{F}$ is a conservative field.
 - (b) Find the potential function of $\vec{F}$.
 - (c) Calculate the work done by $\vec{F}$ from $(0, 1, -1)$ to $(\pi/2, -1, 2)$.

國立虎尾科技大學 108 學年度第二學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第一頁 共一頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 大題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機 且 close book

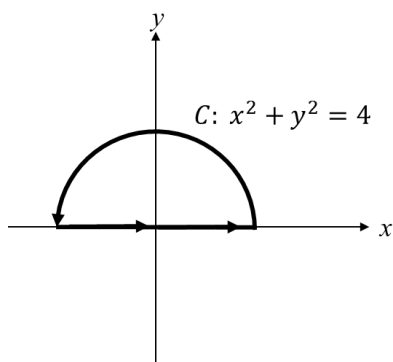
1. Solve I.V.P. $y'' + 4y = 3 \sin(2x)$; $y(0) = 1, y'(0) = 0$.

2. Use Laplace transform to solve I.V.P. $y'' - 2y' + 5y = 1 + t$; $y(0) = 0, y'(0) = 4$.

3. Find the eigenvalues and eigenvectors of the given matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$.

4. Find the half-range cosine and sine expansions of the given function $f(x) = \begin{cases} x, & 0 < x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$.

5. Evaluate $\oint_C (x^2 + y^2)dx - 2xydy$ on the given closed curve C.



國立虎尾科技大學 109 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 1 頁

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) **禁止 使用計算機 close book**

1. Solve I.V.P. $y'' + 2y' = 2x + 5 - e^{-2x}$, $y(0) = 2, y'(0) = 0$.

2. Use Laplace transform to solve I.V.P. $y'' - 4y' = 6e^{3t} - 3e^{-t}$, $y(0) = 1, y'(0) = -1$.

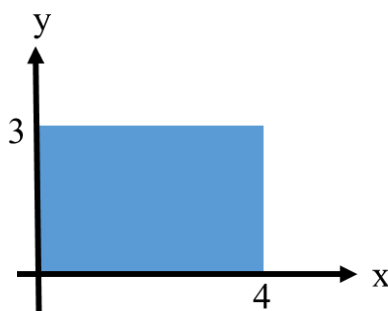
3. Find the eigenvalues and the corresponding eigenvectors of matrix A.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

4. Find the Fourier series of function $f(x)$ on the given interval.

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 \leq x < \pi \end{cases}$$

5. Find the center of mass of the lamina that has the given shape and the area density $\rho(x, y) = xy$.



國立虎尾科技大學 109 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

第 1 頁 共 2 頁

科目：工程數學 (Engineering Mathematics)

注意事項：

- (1) 本試題共有五題，每題註明配分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機 close book and don't use calculator

1. Find solutions of a system of simultaneous linear equations below (30%)

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 4 \\ -x_1 + x_2 + x_3 = 0 \\ 3x_1 - 2x_2 - x_3 = 2 \end{cases}$$

- (1) Please write in standard matrix format $AX=B$ (2) Using Gauss-Jordan Elimination (3) Using reverse Matrix of A (i.e., A^{-1})

2. (a) Please find solution of initial value problem below (20%)

$$\begin{cases} yy'' - (y')^2 = 0 \\ y(0) = \frac{1}{2}, y'(0) = 1 \end{cases}$$

- (b) Please find solution of differential equation below

$$y'' = \frac{1 + (y')^2}{2y}$$

3. (a) Using Laplace transform solve a system of differential equations, (20%)

$$\begin{cases} x' = 2x - 3y \\ y' = -2x + y \end{cases}, x(0) = 8, y(0) = 3$$

- (b) Solve the problem using Laplace transform

$$y'' + 4y' + 4y = 1 + \delta(t - 1), \quad y(0) = 0, y'(0) = 2$$

4. A block of unit mass (with $m=1$) under external force F acting on x -axial, .

$$F = -A\omega^2 \sin(\omega t)$$



As motion starts, this block Q positions on zero location with velocity $A\omega$. Please find Q movement (as function of time). (10%)

5. (a) What is "Fourier Series" used for ? (20%)

(b) Please find Fourier Series

$$f(t) = 1 + t; \quad -1 < t < 1$$

(c) Please find the sum of

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots \dots$$

國立虎尾科技大學 110 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

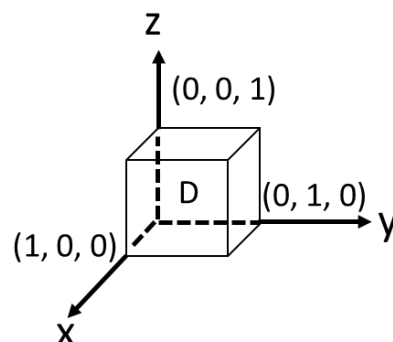
第一頁 共一頁

科目：工程數學 Engineering Mathematics

注意事項：

- (1) 本試題共有五題，每題二十分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機 close book and don't use calculator

1. Solve I.V.P. (a) $y' - 2xy = 2x, y(0) = 1$. (Hint: Linear)
(b) $(x^2 - y)dx + (y - x)dy = 0, y(0) = 1$. (Hint: Exact)
2. Solve I.V.P. $y'' + 4y = 2x, y(0) = 1, y'(0) = 1$.
3. Use the Laplace transfer to solve $y'' - 2y' - 3y = f(t)$, $y(0)=1$, $y'(0)=0$, in which $f(t) = \begin{cases} 0, & t < 4 \\ 3, & t \geq 4 \end{cases}$.
4. Verify the Gauss divergence theorem of the vector field, $\vec{F} = xy\hat{i} + yz\hat{j} + xz\hat{k}$, for the region D bounded by the unit cube defined by $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$.



5. Find the half-range (a) Fourier-cosine (b) Fourier-sine expansions of the function, $f(x)$, on the given interval. $f(x) = x^2 + x, 0 < x < 1$

國立虎尾科技大學 110 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

Page 1 / 1

科目：工程數學

注意事項：

- (1) 本試題共有五題，每題二十分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 禁止使用計算機 close book and don't use calculator

1. (a) Solve $y' + y = e^{-x}, y(0) = 1$. (**Hint: Linear**)
(b) Solve $(x^2 + y)dx + (x - y)dy = 0, y(0) = 2$. (**Hint Exact**)
2. Solve Euler's differential equation: $x^2y'' + 3xy' + 13y = 4 + 3x$ for $x > 0$.
(**Hint: Let $x = e^t$, i.e. $t = \ln(x)$**)
3. Use Laplace transform to solve: $y'' - 4y' = 6e^{3t} - 3e^{-t}, y(0) = 1, y'(0) = -1$.
4. Find the eigenvalues and the corresponding eigenvectors of matrix A.
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
5. (a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where the vector field $\vec{F}$ is $\vec{F} = xy^2\hat{i} + (x^2y + y^3)\hat{j}$ and the curve C is $\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$, $0 \leq t \leq \frac{3\pi}{2}$.
(b) Find the length S of the curve C: $\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + \sqrt{3}t\hat{k}$, $0 \leq t \leq 2\pi$.
(**Hint: $S = \int_C dS = \int_C \left| \frac{d\vec{r}}{dt} \right| dt$**)

國立虎尾科技大學 111 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

Page 1 / 1

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號。
- (3) 可使用計算機，close book, may use calculator,

1. Solve the given differential equation:

$$x^2 y'' + 3xy' + 17y = 4 + 3x$$

2. Find solution of a simultaneous equation, (1) using Gauss-Elimination method; (2) using inverse of matrix method.

$$\begin{cases} 0.3x_1 + 0.52x_2 + x_3 = -0.01 \\ 0.5x_1 + x_2 + 1.9x_3 = 0.67 \\ 0.1x_1 + 0.3x_2 + 0.5x_3 = -0.44 \end{cases}$$

3. Using Laplace transform to solve the IVP problem:

$$y'' - 4y' = 6e^{3t} - 2e^{-t}, y(0)=1, y'(0) = 4.$$

4. Find the Fourier series of $f(x)$ on the given interval:

$$f(x) = \begin{cases} \pi^2, & -\pi < x < 0 \\ \pi^2 - x^2, & 0 \leq x < \pi \end{cases}$$

5. Find the half-range cosine and sine expansions:

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$$

國立虎尾科技大學 111 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

Page 1 / 1

科目：工程數學

注意事項：

- (1) 本試題共有 4 題，每題 25 分，合計一百分。
- (2) 請依序作答於答案卷上並註明題號。
- (3) 可使用計算機， close book , may use calculator ,

1. Solve the given differential equation:

$$x^3 y''' - 3x^2 y'' + 6xy' - 6y = 3 + \ln x^3$$

2. Find solution of a simultaneous equation, (1) using Gauss-Elimination method;
(2) using inverse of matrix method.

$$\begin{cases} 3x_1 - 0.1x_2 - 0.2x_3 = 7.85 \\ 0.1x_1 + 7x_2 - 0.3x_3 = -19.3 \\ 0.3x_1 - 0.2x_2 + 10x_3 = 71.4 \end{cases}$$

3. Using Laplace transform to solve the initial value problem(IVP):

$$y'' - 3y' + 2y = e^{-4t}, y(0)=1, y'(0) = 5 .$$

4. Find the Fourier series of $f(x)$ on the given interval:

$$f(x) = \begin{cases} 1, & -1 < x < 0 \\ x, & 0 \leq x < 1 \end{cases}$$

國立虎尾科技大學 112 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

Page 1/1

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計 100 分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 閉書考、禁止使用計算機
Close book and Engineering calculator is prohibited.

1. Solve I.V.P. $y' + \frac{3}{x}y = \frac{2}{x^2}e^{x^2}$; $y(1) = 0$.

2. Solve I.V.P. $y'' - 4y' + 3y = e^{2x}$; $y(0) = 0, y'(0) = 0$.

3. Use **Laplace transform** to solve I.V.P. $y'' - 3y' + 2y = e^{3t}$; $y(0) = 0, y'(0) = 0$.

4. Find the **inverse (A^{-1})** of matrix A.

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 5 & 2 & 6 \\ 4 & 2 & 4 \end{bmatrix}$$

5. Evaluate $\iint_{\Sigma} f(x, y, z) d\sigma$, where $f(x, y, z) = z$, Σ is the part of the cone $z = \sqrt{x^2 + y^2}$ in the first octant between the plane $z = 2$ and $z = 4$.

國立虎尾科技大學 113 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

Page 1 / 1

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計 100 分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 閉書考、不可使用計算機
Close book and Engineering calculator is not allowable.

1. Solve the given differential equation:

$$y'' + 2y' = 2x + 5 - e^{-2x}$$

2. Solve the Euler's differential equation:

$$xy'' + 3xy' + 13y = 4 + 3x \quad \text{for } x > 0 \quad (\text{Hint: Let } x = e^t)$$

3. Using "Laplace transform" to solve the initial value problem (IVP):

$$y'' + 4y = \delta(t - \pi) - \delta(t - 2\pi) \quad \text{with } y(0) = 0, y'(0) = 1$$

4. Find the eigenvalues and the corresponding eigenvectors of matrix A.

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

5. Find the Fourier series of $f(x)$ on the given interval:

$$f(x) = \begin{cases} 0 & , -1 < x < 0 \\ x & , 0 \leq x < 1 \end{cases}$$

國立虎尾科技大學 114 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

Page 1 / 1

科目：工程數學

注意事項：

- (1) 本試題共有 5 題，每題 20 分，合計 100 分。
- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
- (3) 閉書考、不可使用計算機
Close book and Engineering calculator is not allowable.

1. Solve the given differential equation:

$$y'' + 3y' + 2y = -x + 4 + e^{-3x}$$

2. Solve the Euler-Cauchy differential equation:

$$x^2 y'' - xy' + y = 2, y(1) = 1.5, y'(1) = 0.25 \quad (\text{Hint: Let } y = x^m)$$

3. Using “Laplace transform” to solve the initial value problem (IVP):

$$y'' + 4y = \sin t * u(t - 2\pi) \quad \text{with } y(0) = 1, y'(0) = 0$$

4. Find the eigenvalues and the corresponding eigenvectors of matrix A, and using the eigenvalues and the corresponding eigenvectors to find the inverse of A. (Hint:

$$A = PDP^{-1} \quad \text{and then } A^{-1} = PD^{-1}P^{-1})$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

5. Find the Fourier series of $f(x)$ on the given interval:

$$f(x) = \begin{cases} 1 & , -1 < x < 0 \\ x & , 0 \leq x < 1 \end{cases}$$

立虎尾科技大學 114 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

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科目：工程數學

注意事項：

- (1) 本試題共有 4 題，每題 25 分，合計 100 分。

This test consists of 4 questions. Each question is worth 25 points, for a total of 100 points.

- (2) 請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。

Please answer in order on the answer sheet and clearly indicate the question numbers. If the selected question numbers are not indicated or if more than the required number of questions are answered, only the earlier answers in the response order will be graded.

- (3) 閉書考、可使用計算機。

Close book and Engineering calculator is allowable.

1. Solve the given differential equation:

$$y'' + 5y' + 6y = -x + 1 + 2e^{-x}$$

2. Using “Laplace transform” to solve the initial value problem (IVP):

$$y'' + 4y = \sin t * u(t - 2\pi) \quad \text{with} \quad y(0) = 1, y'(0) = 0$$

3. Find the eigenvalues and the corresponding eigenvectors of matrix A.

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

4. Find the Fourier series of $f(x)$ on the given interval:

$$f(x) = \begin{cases} 0 & , -1 < x < 0 \\ x & , 0 \leq x < 1 \end{cases}$$