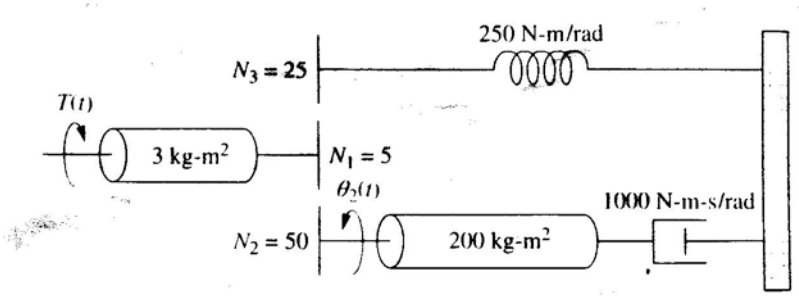
國立虎尾科技大學　機械與機電工程研究所

99 學年度第 1 學期　博士班資格考　【自動控制】

一、如圖所示之旋轉式機械系統，試求轉移函數 $G(s)=\dfrac{\theta_2(s)}{T(s)}$



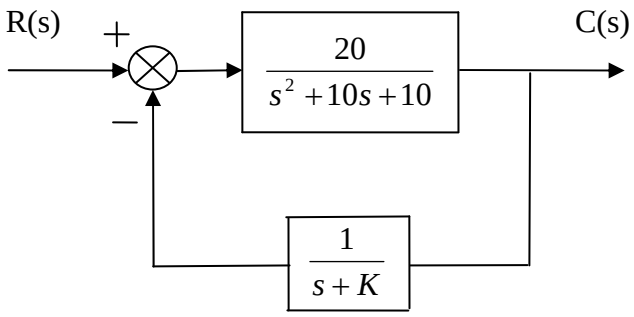
二、狀態方程式如下所示

$$\dot{X}=\begin{bmatrix}0 & 1\\ -3 & -4\end{bmatrix}X$$

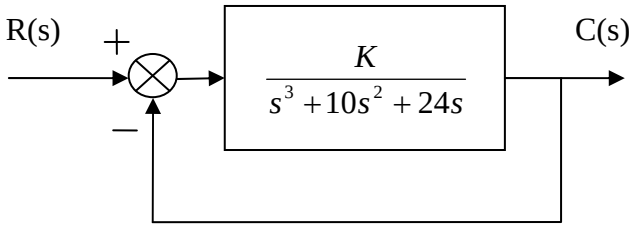
$$X(0)=\begin{bmatrix}1\\ 2\end{bmatrix}$$

試求狀態向量 $X(t)$

三、如圖所示控制系統，系統的輸入爲單位步階，若要求穩態誤差 $e_{ss}=0.25$，請求出 K 值。



四、一單位迴授系統，其轉移函數如下所示，

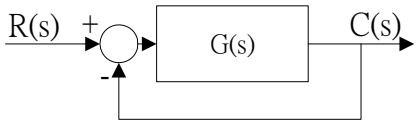


(a) 試繪 $K > 0$ 根軌跡圖

(b) 試求根軌跡與虛軸 $j\omega$ 相交的點和交點的增益 K 值

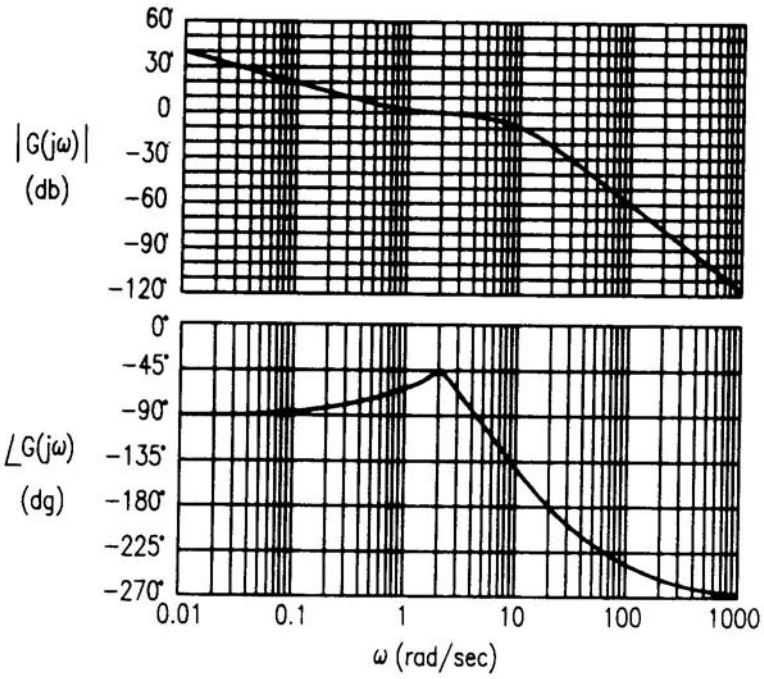
(c) 使系統穩定的 K 值範圍。

五、單位迴授控制系統



Figure

其開迴路轉移函數 G(s) 的波德圖(Bode diagram)如下所示：



(a) 試由波德圖找出增益交越頻率(gain crossover frequency)和相位交越頻率(phase crossover frequency)

(b) 試由波德圖找出增益邊限(gain margin)和相位邊限(phase margin)

(c) 判斷此系統是否穩定(需簡要說明理由)

國立虎尾科技大學 106 學年度第一學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班　　第 1 頁　共 2 頁

科目：自動控制

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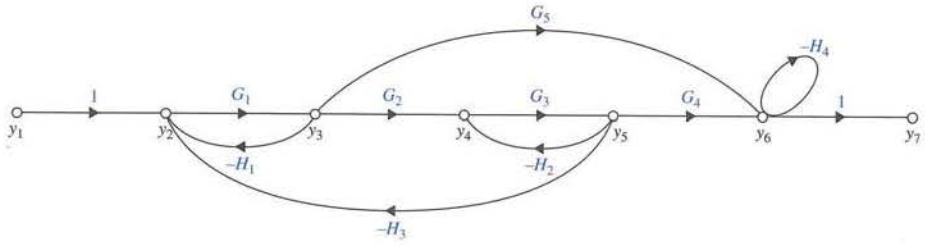
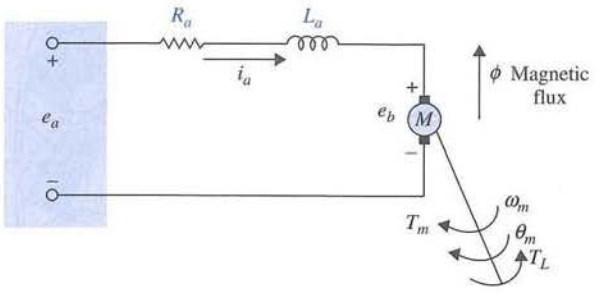
（3）可 使用計算機 close book

1. 試求下圖訊號流圖(Signal-Flow Graph)之輸出與輸入間的增益。



2. 試求下圖之分激式直流馬達(Separately Excited DC Motor)的狀態方程式(State Equations)與馬達位移輸出與輸入電壓間的轉移函數(Transfer Function)。



3. 試求下列狀態方程式的控制性標準型(Controllability Canonical Form)：

$$\frac{dx(t)}{dt}=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}x(t)+\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}u(t)$$

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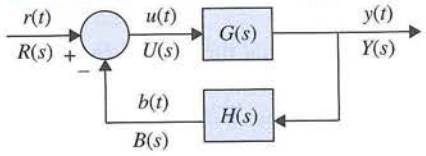
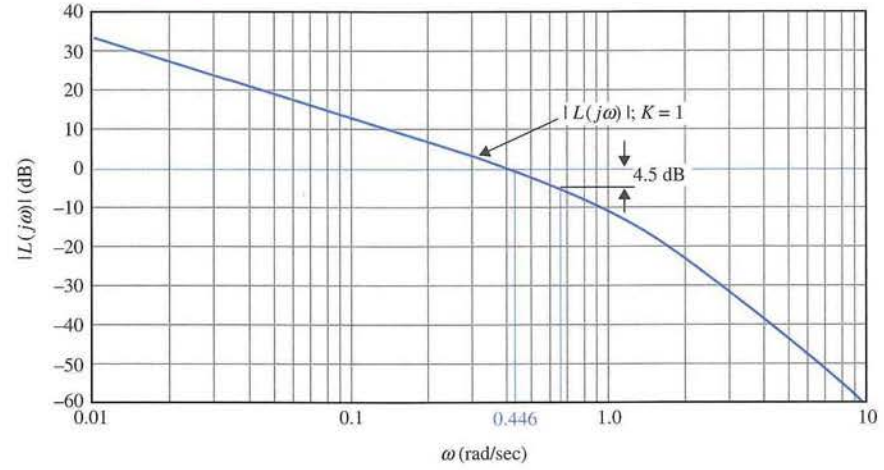
系別：動力機械工程系機械與機電工程博士班　　第 2 頁　共 2 頁

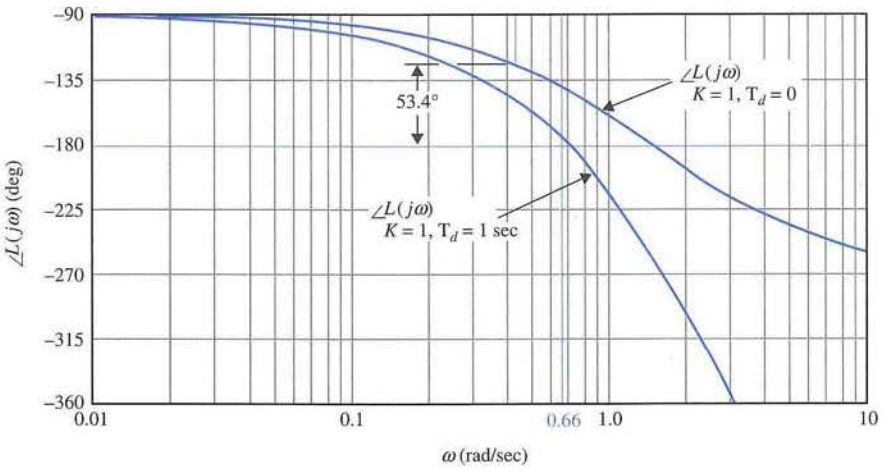
科目：自動控制

4. 下圖回授控制系統中，轉移函數 $G(s)=\dfrac{1}{s^2(s+12)}$，$H(s)=\dfrac{10s}{s+5}$。試求單位步階輸入(Unit-Step Input)時的穩態誤差(Steady-State Error)。



5. 一閉迴路系統(Closed-Loop System)的迴路轉移函數(Loop Transfer Function)為 $L(s)=\dfrac{100K(s+5)(s+40)}{s^3(s+100)(s+200)}$，其 $K=1$ 時 $L(j\omega)$ 的波德圖(Bode Plot)如下圖所示。試求系統穩定時之增益交越點頻率(Gain-Crossover Frequency)與相位邊限(Phase Margin)。





國立虎尾科技大學 106 學年度第二學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班　　第一頁　共二頁

科目：自動控制

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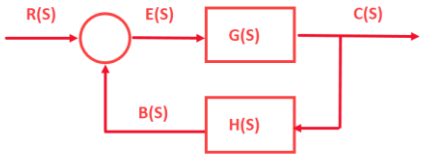
（3）可使用計算機 close book

第一題 (自控基本類型題):



1. 求此閉迴路之轉移矩陣 M 與 G、H 關係 find relation among transfer matrix M,G and H
2. 設 G 與 H 如下，求出轉移矩陣 M　　G and H given below, please find M(s)

$$G(s)=\begin{bmatrix}\dfrac{1}{s+1} & -\dfrac{1}{s}\\ 2 & \dfrac{1}{s+2}\end{bmatrix} \qquad H(s)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

第二題 (數學基本功夫):

狀態方程式如下 Consider the state equation

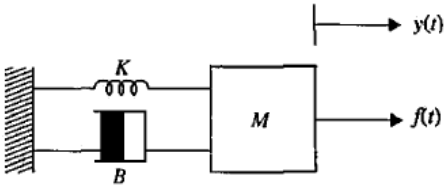
$$\begin{bmatrix}\dfrac{dx1(t)}{dt}\\ \dfrac{dx2(t)}{dt}\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}dx1(t)\\ dx2(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

求解狀態轉移矩陣 (please determine state-transition matrix) Ø(t)

第三題 (數學模式:機):

(1) Please find the state equation (first order) on this mechanical system (2) the transfer function between Y(s) and F(s)

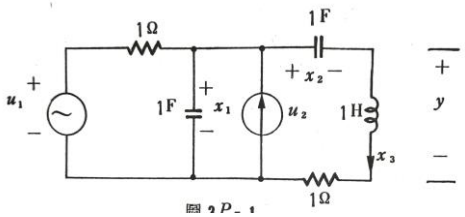
求下圖所示機械系統之(1)狀態方程式、(2) Y(s)F(s)間的轉移函數



第四題 (數學模式:電):

Electric network u1 is voltage input and u2 us current source . Y is voltage output (1) find state equation of the circuit (2) state-transition matrix

求下圖所示電子迴路，有兩個輸入u_1為電壓源、u_2為電流源。電感二端電壓 y 為輸出



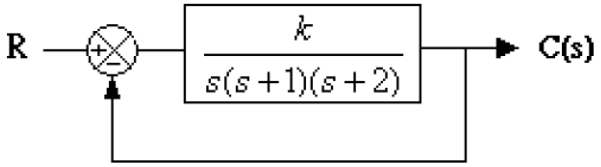
試求: (a) 迴路的動態方程式 (b)轉移函數矩陣

第五題 (阻尼特性):

單位回授控制系統的開環路轉移函數 system transfer function $G(s)=\frac{K}{s(s+1)(s+2)}$ $K>0$ 求

(a) 系統具有臨界阻尼特性 K 的值? Please find K to make this system critically damped

(b) 系統具阻尼比為 0.5 特性 K 的值? Please find K to have damping ratio 0.5



第六題 (根軌跡圖) Root Locus Analysis

開迴路轉移函數$G(s)=\frac{K}{(s+4)(s+2)^2}$ ，試問點$s=-2+3j$ 及 s=-1.33+2j 是否位於系統閉環路的根軌跡上。需要繪圖說明。Please show if $s=-2+3j$ and s=-1.33+2j are located in root loci. Please show on the root loci plots.

第七題 (根軌跡圖) Root Locus Analysis

開迴路轉移函數$G(s)=\frac{K}{s(s+3)(s^2+2s+2)}$ ，試求下列數值。

1. 根軌跡之漸進線與時數軸之交叉點及漸進線趨向無窮處之角度 The intersection point of the progressive line of the root trajectory (root loci) with the time axis and the angle of the progressive line toward infinity
2. 求出此系統達到臨界穩定時之 K 值。Please find K to have system reach system critical stability
3. 匯出此系統之根軌跡圖。Root locus diagram

國立虎尾科技大學 107 學年度第一學期博士班資格考試題

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科目：自動控制

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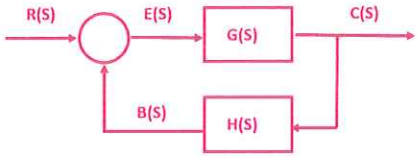
（2）請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。

（3）可使用計算機、閉書考 hand calculator is allowed， close book test

第一題 **(自控基本類型題):**



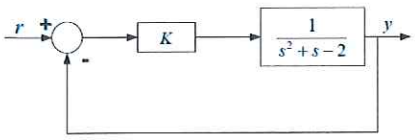
1. 求此閉迴路之轉移矩陣 M 與 G、H 關係 find relation among transfer matrix M,G and H
2. 設 G 與 H 如下，求出閉迴路轉移矩陣 M　　G and H given below, please find M(s)

$$G(s)=\frac{1}{s^3+s+1}\quad H(s)=\frac{3}{s+3}$$

3. 利用羅斯穩定準則判斷此閉迴路系統的穩定性？Use the Routh-criterion to judge the stability of this closed loop system

第二題 **(數學基本功夫):**

若定義 $x_1=y$，$x_2=\dot{y}$，系統狀態空間表示式可表示為:



$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=A\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+Br$$

$$y=C\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

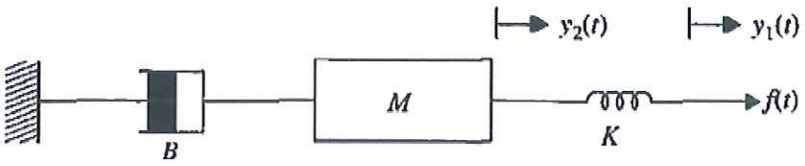
，試求:

Please find (1) 狀態空間可控制典型式(Controllable Canonical Form)下的 A , B , C (2) A 矩陣之特徵值 A matrix eigen value (3) 由(2)之結果，試求 K 之範圍保證系統穩定 find K to ensure stability (4)使用羅斯表方法驗證(3)的結果 Use the Routh-criterion to judge (3) results

第三題 **(數學模式:機): (第三題、第四題任選一題作答)**

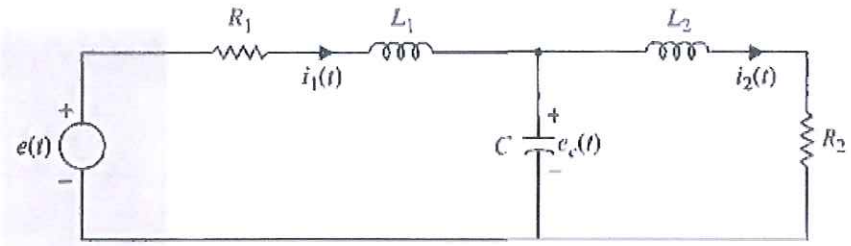
(1) Please find the state equation (first order) on this mechanical system (2) the transfer function between Y(s) and F(s)

求下圖所示機械系統之(1)狀態方程式、(2) y(s)f(s)間的轉移函數



第四題 **(數學模式:電): (第三題、第四題任選一題作答)**

RLC Electric network (1) find state equation of the circuit in matrix form (2) signal flow graph representation (3) The transfer functions between $I_1(s)$ and E(s), $I_2(s)$ and E(s), and Ec(S) and E(s), respectively 求下圖所示電子迴路



試求: **(1)** 求狀態方程式向量-矩陣形式 **(2)** 繪製狀態圖 **(signal flow graph representation)** **(3)**轉移函數 $I_1(s)$ and E(s), $I_2(s)$ and E(s), and Ec(S) and E(s)

第五題 **(阻尼特性):**

單位回授控制系統的開環路轉移函數 system transfer function $G(s) = \frac{K}{s(s+1)(s+2)}$ $K > 0$ 求

(a) 系統具有臨界阻尼特性 K 的值? Please find K to make this system critically damped

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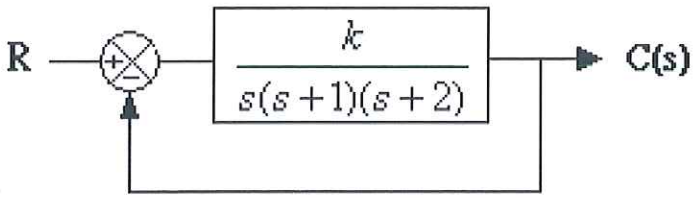
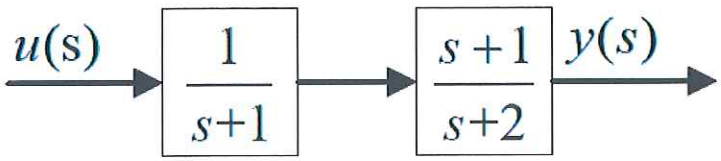


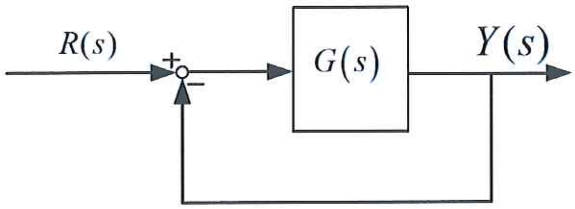
第六題

考慮下圖，以二階狀態空間描述之系統是否為可控?是否為可觀? Show below system's controllability and observability



第七題 **Gain margin problem**

考慮閉迴路系$G(s)=\frac{1}{s(1+0.5s)(1+s)}$，求出相位邊限(Gain margin)。Find Gain margin for a given G(s)。



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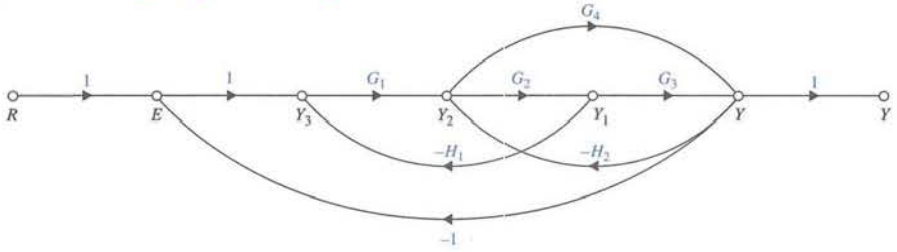
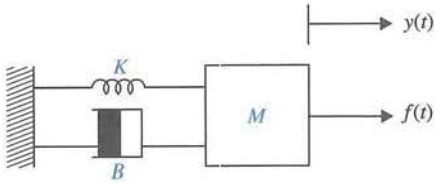
（3）可使用計算機 close book

1. The signal-flow graph of a system is



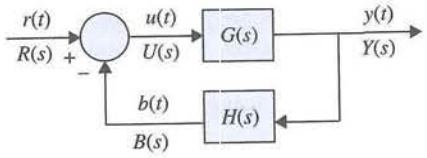
Find the transfer function of the system ($\frac{Y(s)}{R(s)}$).

2. Consider the mass-spring-friction system



Find the transfer function between $Y(s)$ and $F(s)$ with zero initial conditions.

3. Consider the system



which has the following transfer functions:

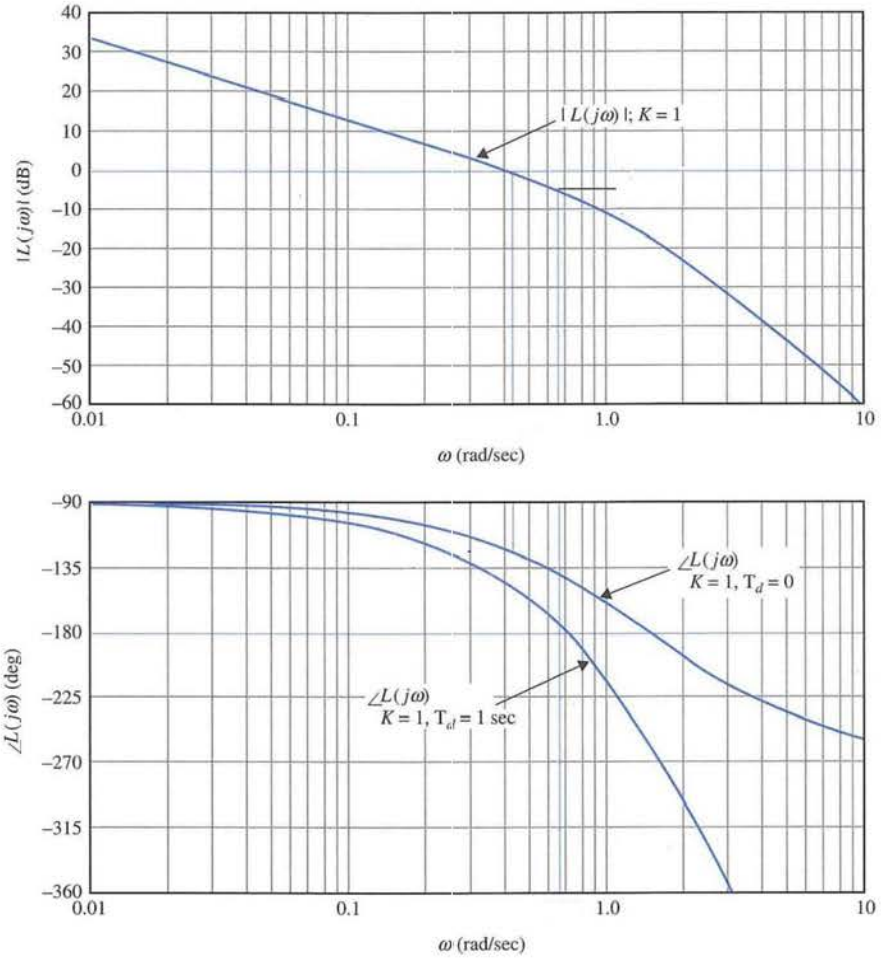
$$G(s) = \frac{1}{s^2(s+12)} \qquad H(s) = \frac{5(s+1)}{s+5}$$

Find the steady-state errors of the system for the unit-step input, the unit-ramp input and the unit-parabolic input respectively.

4. Consider that the loop transfer function of a closed-loop system is

$$L(s) = \frac{Ke^{T_d s}}{s(s+1)(s+2)}$$

The Bode plot of $L(jw)$ with $K = 1$ and $T_d = 0$ is shown as



Find the gain margin, phase margin, gain-crossover frequency and phase-crossover frequency.

國立虎尾科技大學 109 學年度第 1 學期博士班資格考試題

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（3）可使用計算機與 close book

1. The following differential equations represent linear time-invariant systems. Write the dynamic equations (state equations and output equations) in vector-matrix form.

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + y(t) = 5r(t)$$

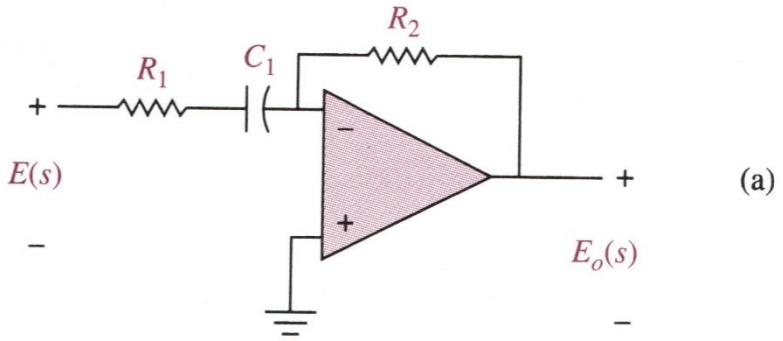
2. The state equations of a linear time-invariant system are represented by

$$\frac{dx(t)}{dt} = \boldsymbol{Ax}(t) + \boldsymbol{Bu}(t)$$

Find the state transition matrix ϕ(t), the characteristic equation, and the eigenvalues of A for the following cases.

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

3. Find the transfer functions E_o(S)/E(s) for the circuits shown in Fig.(a)



(a)

4. Find the poles and zeros of the following functions (including the ones at infinity, if any). Mark the finite poles with X and the finite zeros with O in the s-plane.

$$F(s)=\frac{5(s+2)}{s(s+1)(s+3)^2}$$

5. Find the Laplace transforms of the following functions. Use the theorems on Laplace transforms if applicable.

$f(t)=3+5e^{-7t}+2t^3$, (t>0)

國立虎尾科技大學 109 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班　　第 1 頁　共 2 頁

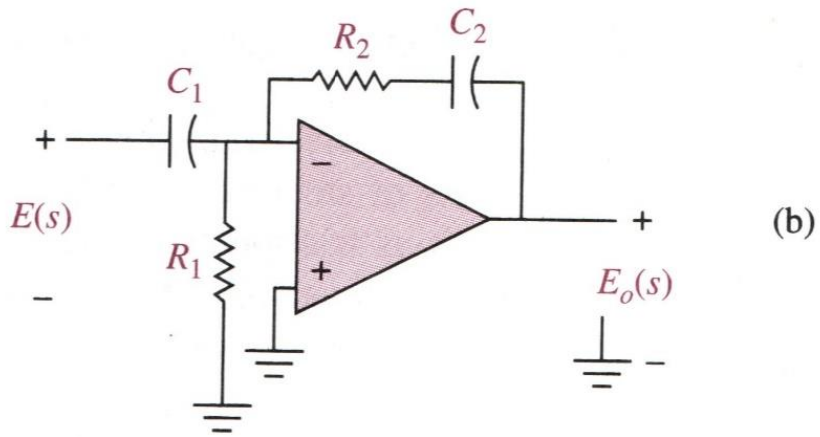
科目：自動控制

注意事項：
（1）本試題共有 5 題，每題 20 分，合計一百分。
（2）請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
（3）可使用計算機與 close book

1. Solve the following differential equations by means of the Laplace transform. Assume zero initial conditions.

$$\frac{d^2f(t)}{dt^2} + 5\frac{df(t)}{dt} + 4f(t) = e^{-2t}u_s(t)$$

2. Find the transfer functions $E_0(s)/E(s)$for the circuits shown in Fig.(b)



(b)

3. For each of the characteristic equations of feedback control systems given, determine the range of K so that the system is asymptotically stable. Determine the value of K so that the system is marginally stable and the frequency of sustained oscillation if applicable.

$$s^3 + 20s^2 + 5s + 10K = 0$$

4. The state equations of a linear time-invariant system are represented by

$$\frac{dx(t)}{dt} = \boldsymbol{Ax}(t) + \boldsymbol{Bu}(t)$$

Find the state transition matrix ϕ(t), the characteristic equation, and the eigenvalues of A for the following cases.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

5. Find the stead-state errors of the following single-loop control system for a unit-step input, a unit-ramp input, and a parabolic input, $(t^2/2)u_s(t)$. For system that include a parameter K, find its value so that the answers are valid.

$$\mathrm{M(s)} = \frac{s+1}{s^4+16s^3+48s^2+4s+4} \qquad K_H = 1$$

國立虎尾科技大學 110 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班　　第 1 頁　共 3 頁

科目：自動控制 Automatic Control

注意事項：

（1）本試題共有 10 題，每題 10 分，合計一百分。

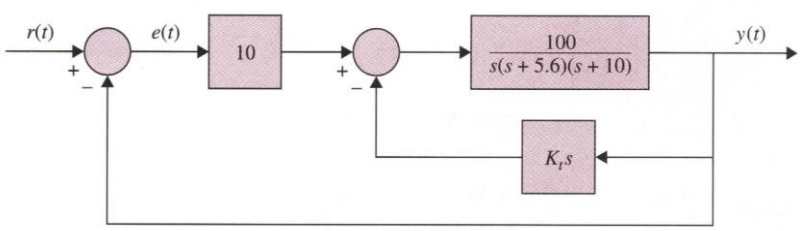
（2）請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。

（3）禁止/可 使用計算機 close book and don't use calculator

1. Determine the step, ramp, and parabolic error constants of the following unity-feedback control systems. The forward-path transfer functions are given.

$$G(s) = \frac{1000}{(1 + 0.1s)(1 + 10s)}$$

2. The block diagram of a motor-control system with tachometer feedback is shown in the Figure. Find the range of the tachometer constant K_t, so that the system is asymptotically stable.



3. The state equations of a linear time-invariant system are represented by

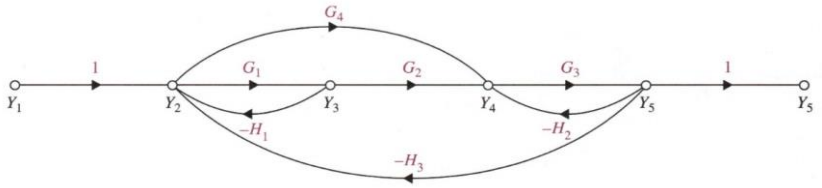
$$\frac{dx(t)}{dt} = \boldsymbol{Ax}(t) + \boldsymbol{Bu}(t)$$

Find the state transition matrix ϕ(t), the characteristic equation, and the eigenvalues of A for the following cases.

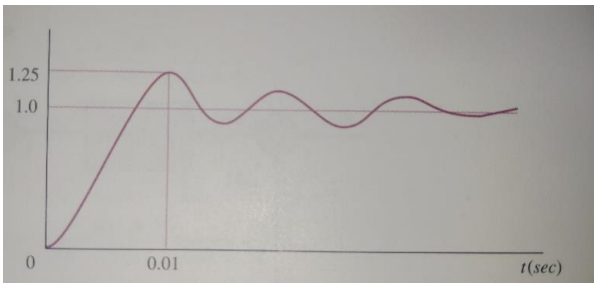
$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

4. Apply the gain formula to the SFGs shown in the Figure. find the following transfer functions:

$$\frac{Y_5}{Y_1}$$

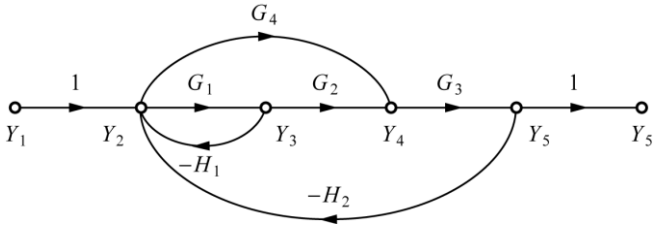


5. The unit-step response of a linear control system is shown in the Figure. Find the transfer function of a second-order prototype system to model the system.

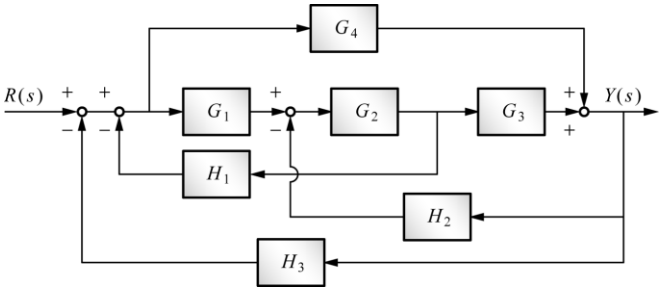


6. Find the following transfer functions in the Figure：

$$\frac{Y_5}{Y_1}$$



7. Find the following transfer functions in the Figure：$Y(s)/R(s)$



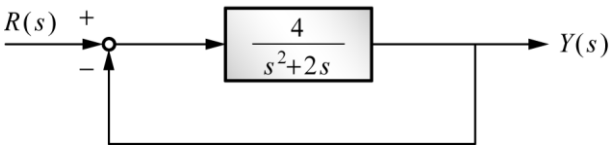
8. For each of the characteristic equations of feedback control systems given, determine the range of K so that the system is asymptotically stable. Determine the value of K so that the system is marginally stable and the frequency of sustained oscillation if applicable.

$s^3+5s^2+(K-6)s+K=0$

9. The following transfer functions are given for a single-loop nonunity-feedback control system. Find the steady-state errors due to a unit-step input, a unit-ramp input, and a parabolic input, $(t^2/2)u_s(t)$.

$$G(s) = \frac{1}{s^2(s+12)} \qquad H(s) = 5(s+2)$$

10. In the Figure. Find (a)rising time (b) peak overshoot M_p (c) damping ratio ζ (d) natural undamped frequency ω_n (e) damping factor α (f) damping frequency ω_d



國立虎尾科技大學 113 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班　　第 1 頁　共 2 頁

科目：自動控制

注意事項：

（1）本試題共有 5 題，每題 20 分，合計一百分。

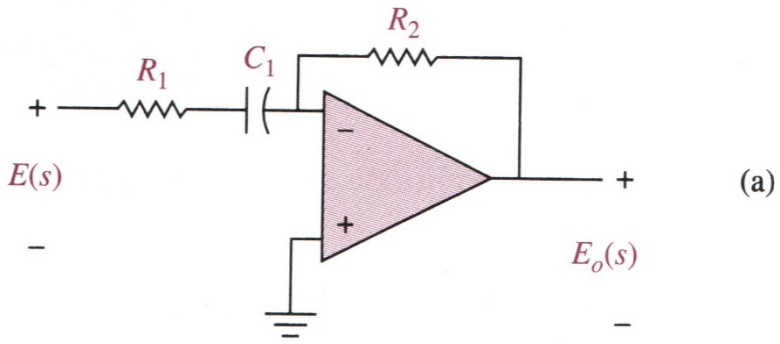
（2）請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。

（3）可使用計算機與 close book

1. The following differential equations represent linear time-invariant systems. Write the dynamic equations (state equations and output equations) in vector-matrix form.

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + y(t) = 5r(t)$$

2. Find the transfer functions $E_0(s)/E(s)$for the circuits shown in Fig.(b)



(a)

3. The state equations of a linear time-invariant system are represented by

$$\frac{dx(t)}{dt} = \boldsymbol{Ax}(t) + \boldsymbol{Bu}(t)$$

Find the state transition matrix ϕ (t), the characteristic equation, and the eigenvalues of A for the following cases.

$$A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

4. For each of the characteristic equations of feedback control systems given, determine the range of K so that the system is asymptotically stable. Determine the value of K so that the system is marginally stable and the frequency of sustained oscillation if applicable.

$$s^3 + (K+2)s^2 + 2Ks + 10 = 0$$

5. The block diagram of a motor-control system with tachometer feedback is shown in Fig.20. Find the range of the tachometer constant K_t, so that the system is asymptotically stable.

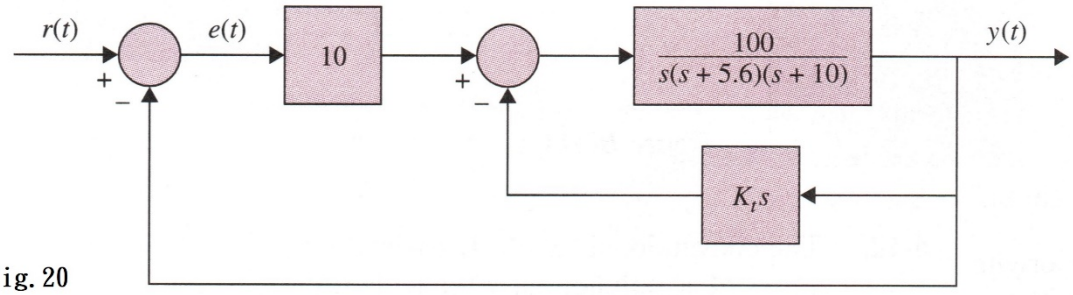


Fig. 20

國立虎尾科技大學 114 學年度第 1 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

科目：自動控制

注意事項：

（1）本試題共有 4 題，每題 25 分，合計 100 分。
This test consists of four questions. Each question is worth 25 points, for a total of 100 points.

（2）請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
Please answer in order on the answer sheet and clearly indicate the question numbers. If the selected question numbers are not indicated or if more than the required number of questions are answered, only the earlier answers in the response order will be graded.

（3）閉書考、可使用計算機
Close book and Engineering calculator is allowable.

1. Find the Laplace transforms of the following functions. Use the theorems on Laplace transforms if applicable.

   g(t)=$5te^{-5t}u_s(t)$

2. Solve the following differential equations by means of the Laplace transform. Assume zero initial conditions.

$$\frac{d^2f(t)}{dt^2} + 5\frac{df(t)}{dt} + 4f(t) = e^{-2t}u_s(t)$$

3. For each of the characteristic equations of feedback control systems given, determine the range of K so that the system is asymptotically stable. Determine the value of K so that the system is marginally stable and the frequency of sustained oscillation if applicable.

$$s^3 + (K+2)s^2 + 2Ks + 10 = 0$$

4. The block diagram of a motor-control system with tachometer feedback is shown in Fig.20. Find the range of the tachometer constant K_t, so that the system is asymptotically stable.

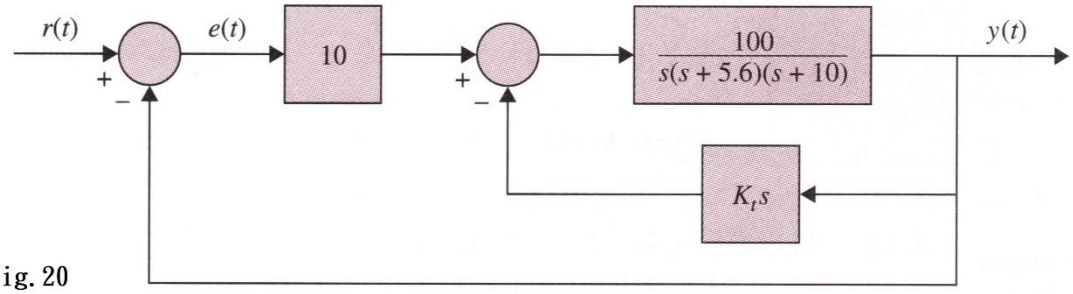


Fig. 20

國立虎尾科技大學 114 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

科目：自動控制

注意事項：

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This test consists of 4 questions. Each question is worth 25 points, for a total of 100 points.

（2）請依序作答於答案卷上並註明題號，若未註明選答題號及超過規定題數時，謹採計作答順序較前之題目計分。
Please answer in order on the answer sheet and clearly indicate the question numbers. If the selected question numbers are not indicated or if more than the required number of questions are answered, only the earlier answers in the response order will be graded.

（3）閉書考、可使用計算機。
Close book and Engineering calculator is allowable.

1. For each of the characteristic equations of feedback control systems given, determine the range of K so that the system is asymptotically stable. Determine the value of K so that the system is marginally stable and the frequency of sustained oscillation if applicable.

$$s^4 + 25s^3 + 15s^2 + 20s + K = 0$$

2. Determine the step, ramp, and parabolic error constants of the following unity-feedback control systems. The forward-path transfer functions are given.

$$G(s) = \frac{1000}{S(S+10)(S+100)}$$

國立虎尾科技大學 114 學年度第 2 學期博士班資格考試題

系別：動力機械工程系機械與機電工程博士班

科目：自動控制

3. Consider the signal flow graph of Fig(a). Find the following transfer functions: Y(s)/R(s)

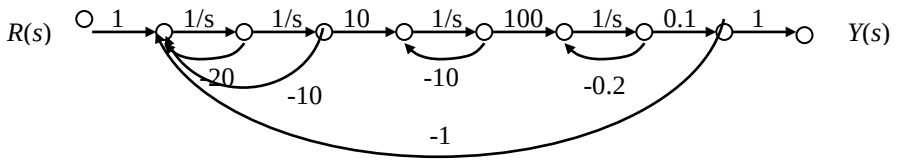

Fig. (a)

4. Find the inverse Laplace transforms of the following functions.

$$F(s)=\frac{2}{s(s+1)(s+2)}$$